Concrete Design

Formwork
All formwork shall be of sufficient strength to carry all construction loads and the joints shall be sufficiently tight to prevent leakage of mortar.

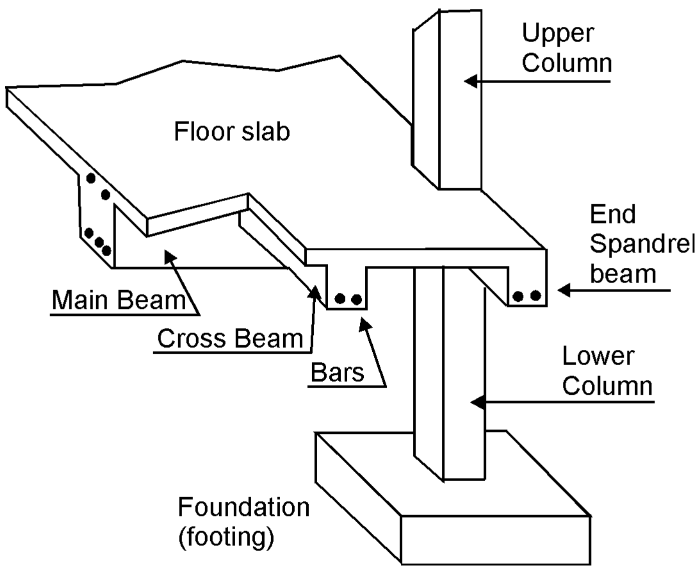
Inspection and cleaning of formwork
Adequate space shall be provided in the forms of columns, walls or deep beams to enable inspection and cleaning. All loose material must be removed.

Removal of formwork.
Formwork may only be removed when the structure has developed enough strength to maintain the integrity of the structure. Cube tests are used to determine the concrete strength. The minimum period that shall elapse between mixing of the concrete and the removal of forms and shoring shall be as follows:

1	2	3
Positioning of shuttering	Minimum time, days, before stripping	
	When using ordinary port-land cement	Use using rapid-hardening Portland cement
Sides of beams, columns, and walls	3	2
Soffits of slabs	10	5
Soffits of beams	21	10

These times shall be increased by the time during which no concrete may be placed; such as during times of falling temperature and extremely cold weather.

Concrete Structural Systems



Typical Beam/column/slab

Floor slabs
Transmit live loads as well as stationary dead loads to the horizontal beams, which are part of the frame. Slabs do this in bending and shear.

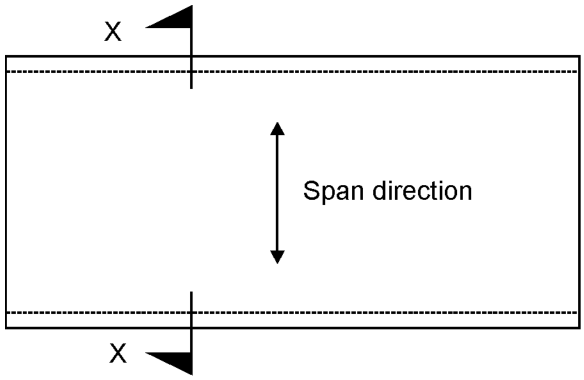
Beams
Transmit loads from the slabs to the vertical columns in bending and shear.

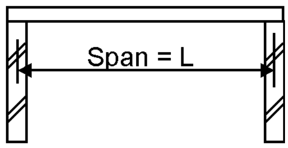
Columns
Columns carry all vertical loads to the foundations in compression. There may be bending moments as a result of the frame action.

Foundations
Carry all vertical loads to the soil. Vertical loads include all dead load and live loads.

Slabs:

Slabs may be simply supported and span in one or two directions.

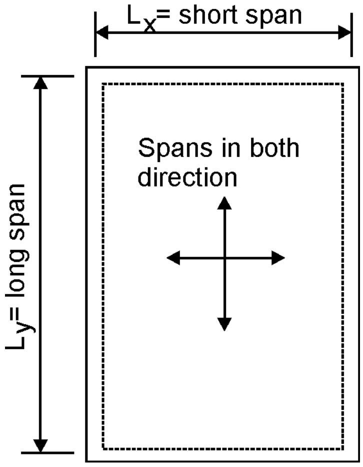




Section XX

Simply Supported Slab Spanning In One Direction

In this case one would look at a 1 m wide strip. The bending moment in the middle (maximum) and the shear force at the supports (Maximum) would be $M = \frac{w \cdot L^2}{8}$ *and* $V = \frac{w \cdot L}{2}$. **Typical thickness of the slab would be span/20 – span/25.**



Concrete Slab Supported on Four Sides.

If Ly/Lx $\leq$ 2 the slab spans in two directions as shown in the sketch

If Ly/Lx > 2 the slab spans only in the shorter direction.

The span direction will determine the bending moment in that direction as well as the vertical load that is transferred to the supports. The following table gives the proportion of the bending moment for various Ly/Lx values.

$$M_X = \alpha_X \cdot w \cdot L_X^2$$
$$M_Y = \alpha_Y \cdot w \cdot L_X^2$$

Ly/Lx	1,0	1,1	1,2	1,3	1,4	1,5	1,75	2,0	2,5	3,0
α_X	0,062	0,074	0,084	0,093	0,099	0,104	0,133	0,118	0,122	0,124
α_Y	0,062	0,061	0,059	0,055	0,051	0,046	0,037	0,029	0,020	0,014

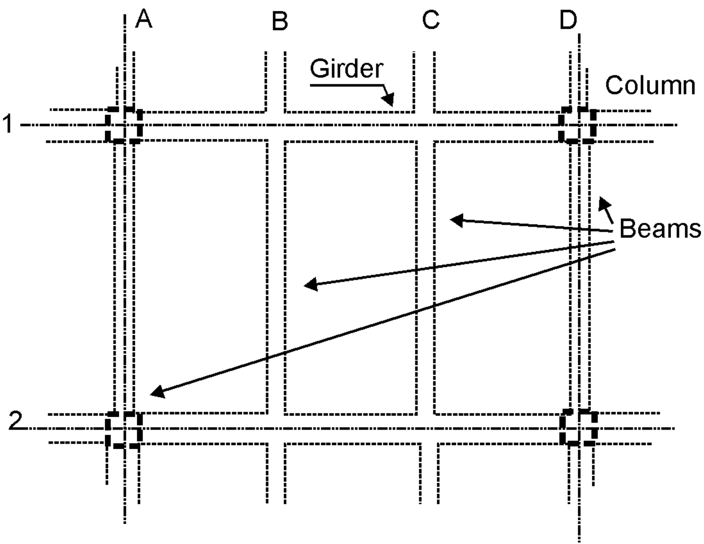
Table: Bending moment coefficients for slabs spanning in two directions at right angles, simply supported on four sides.

These values are based on a unit wide strip in both directions that are bound to the same deflection.

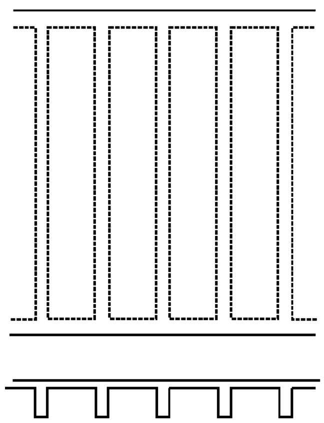
Typical thickness of slab that spans in two directions would be, **Span/30 and as the Ly/Lx proportion gets bigger it would tend towards Span/25.**

One-Way Continuous Slabs And Beams

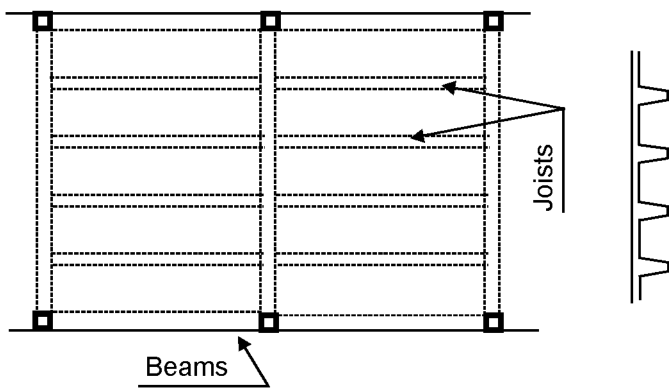
The following sketches show some one-way spanning continuous slabs and beams.



Beams and girders



Section Through Slab

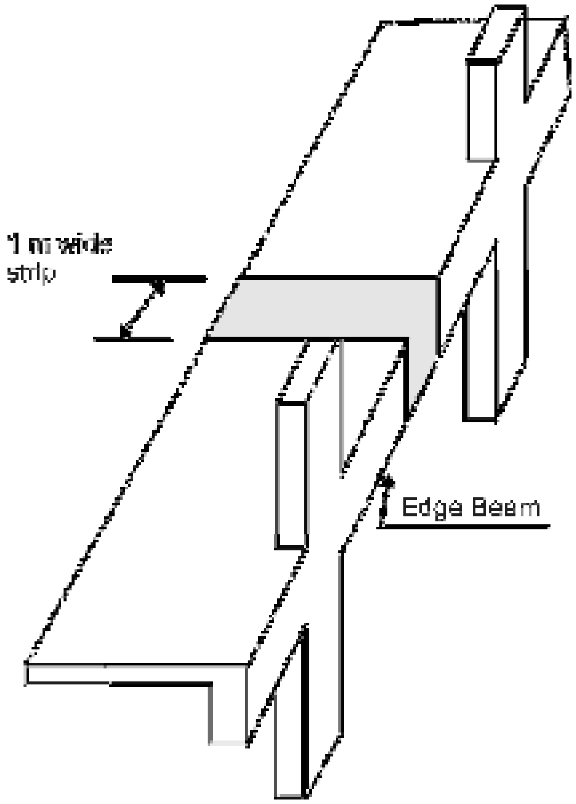


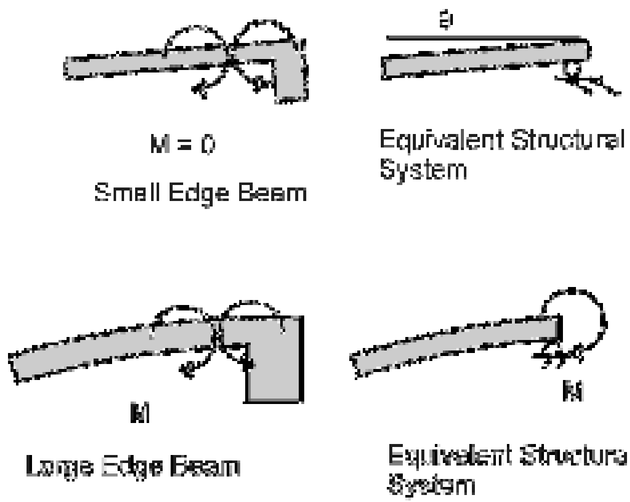
T-beam and slab

One-way joist and girder

One-way structural systems

Remember that restraints may be provided by the edge beams:

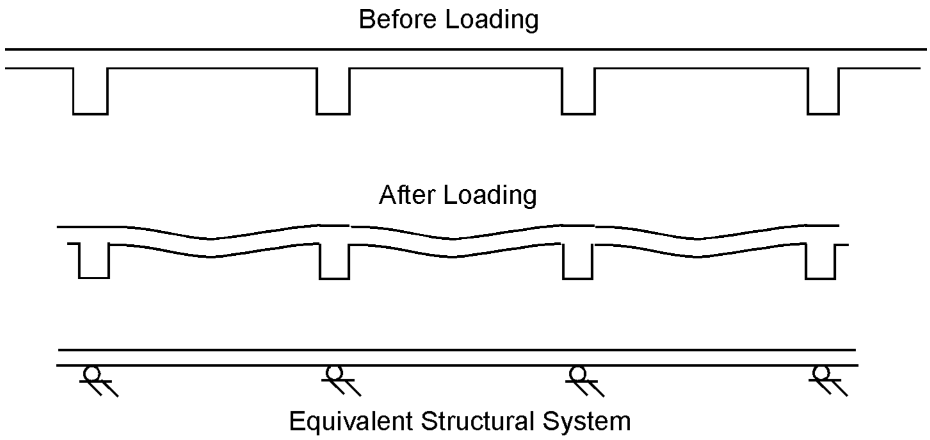




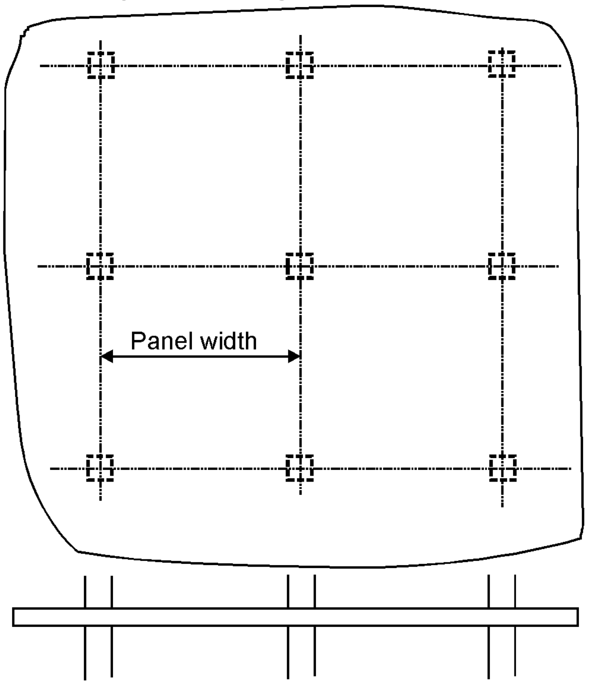
Restraint offered by edge beams

Continuous Flooring Systems

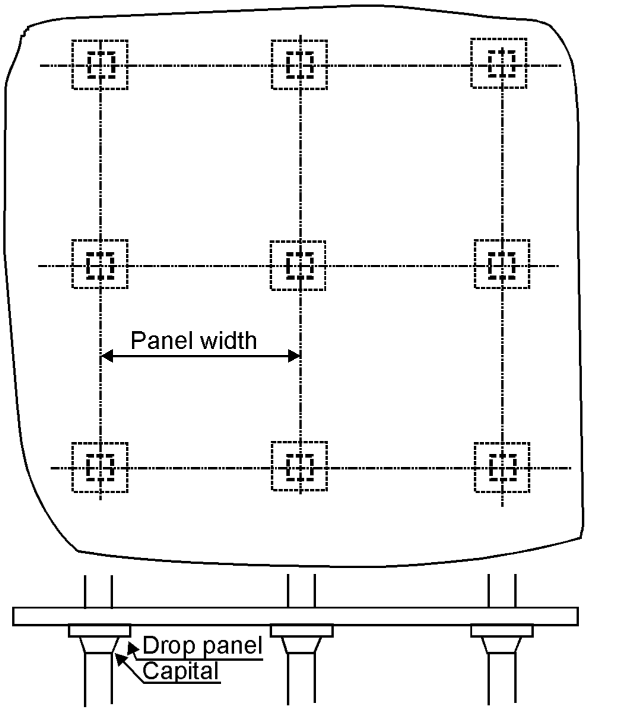
The beam system may be one-way but the floor slab may be continuous.



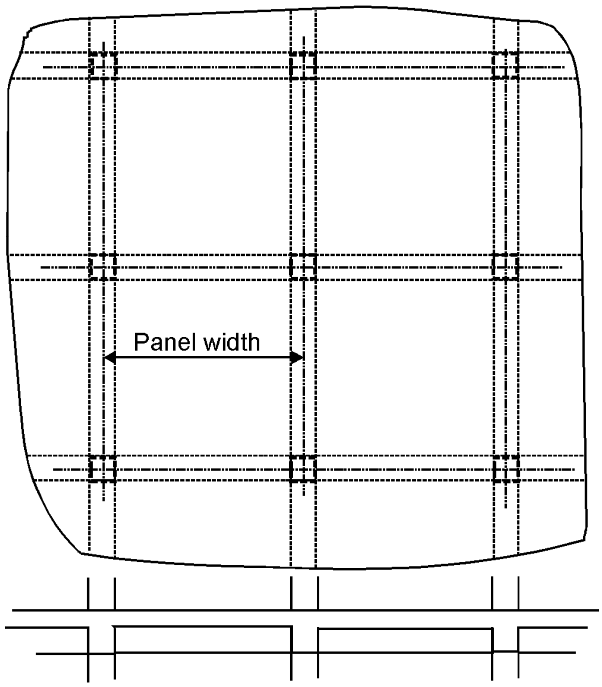
Two-way Spanning Slabs



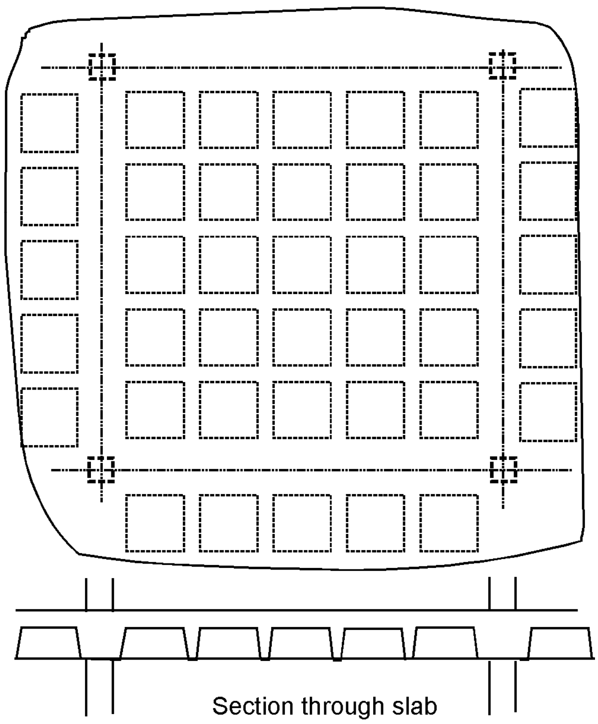
Section through slab



Section through slab

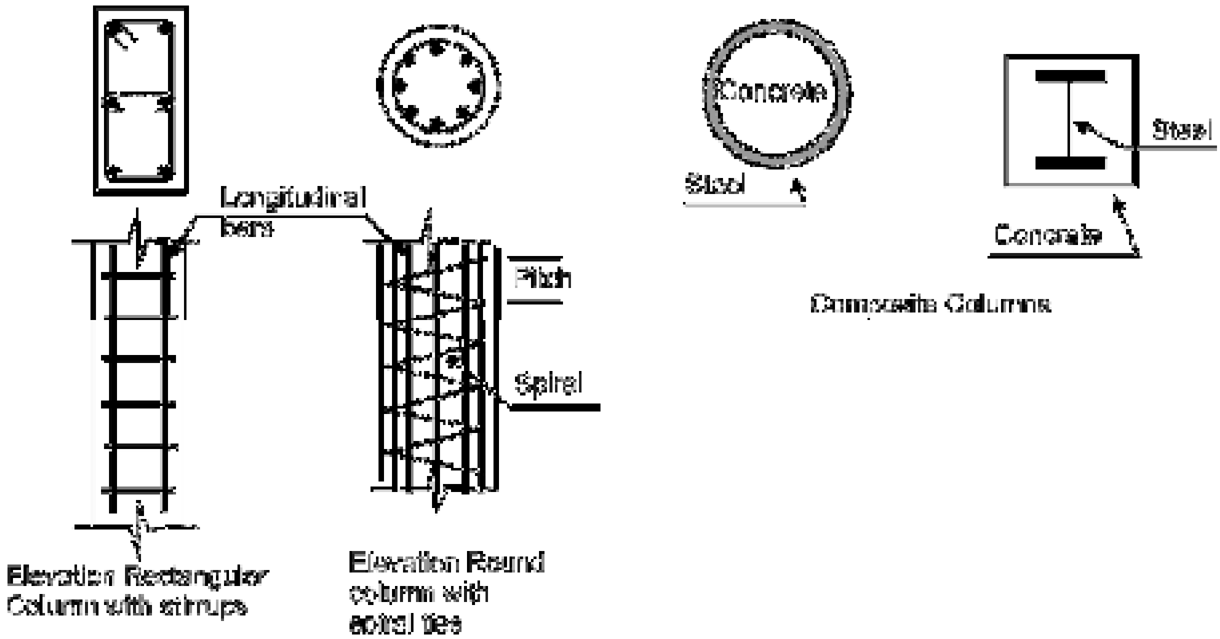


Section through slab

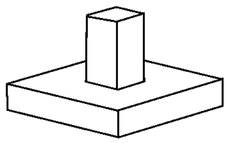


Section through slab

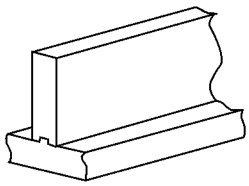
Columns:



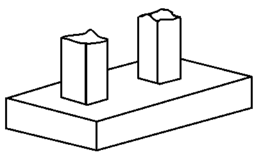
Footings:



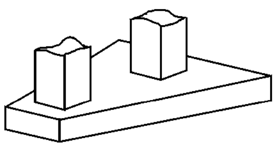
Individual Column Footing



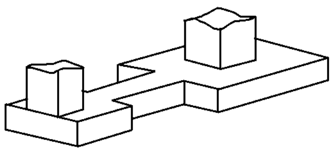
Wall Footing



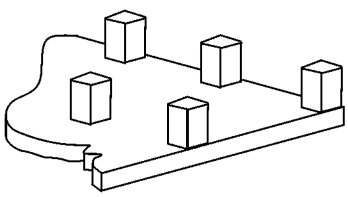
Combined Footing (Rectangular)



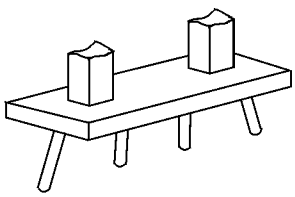
Combined Footing (Trapezoidal)



Combined Footing (Strap or Cantilever)

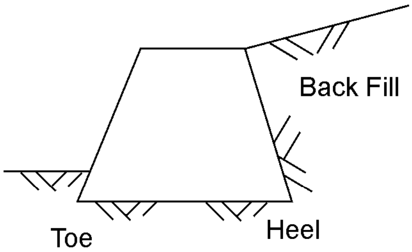


Mat Footing

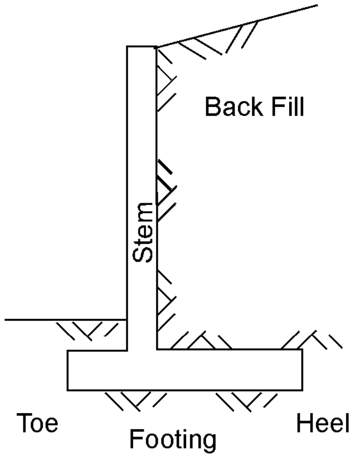


Pile Footing

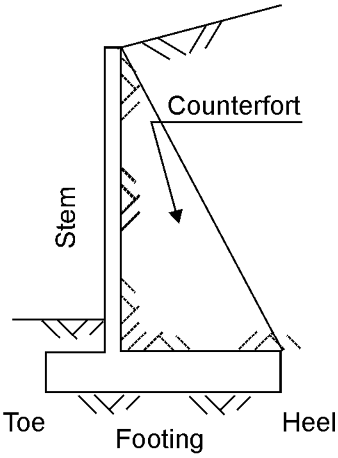
Wall Types

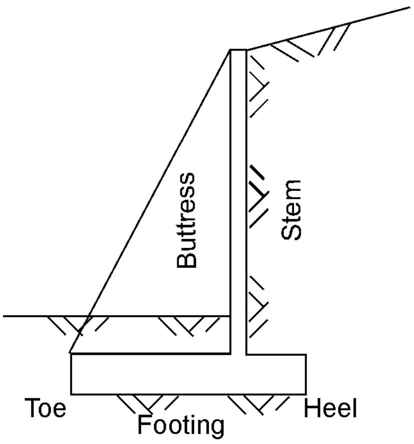


Gravity Wall

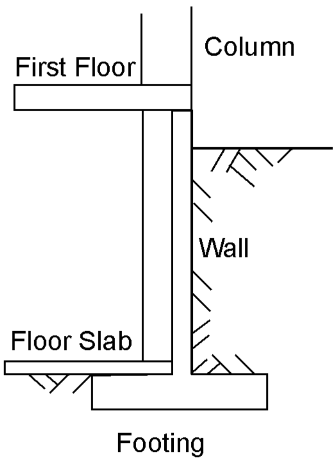


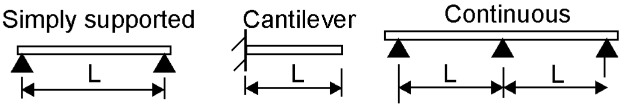
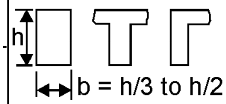
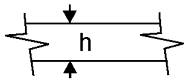
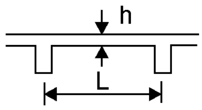
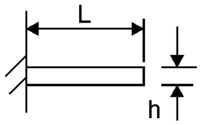
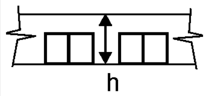
Cantilever Wall





Buttress Wall



Preliminary Planning and Design of Building Structures Approximate initial depth, h, of reinforced concrete slabs and beams			
Structural Element Simply supported / Cantilever / Continuous 	Typical section	Approximate initial depth h	Approximate maximum recommended span
Beam Cast in- situ Simply Supported		L/16	12 m
Continuous – One end continuous		L/18	16 m
Continuous – Both ends continuous		L/21	
Cantilever		L/8	3 - 4 m Be very careful
Solid slabs without ribs Simply supported, one-way span		L/20	6 m
Continuous – One end continuous		L/24	7,5 m
Continuous – Both ends continuous		L/28	
Continuous one-way, supported by beams on either side cast monolithically with slab		L/30	8 - 9 m
Simply supported, two-way span (not flat slab)		Short span / 28	7 m
Continuous, two-way span (not flat slab)		Short span / 32	8 m
Continuous two-way, supported by beams on either side cast monolithically with slab		Short span / 35	9 - 10 m
Cantilever slab		L/10	2 - 3 m Be very careful
Hollow block floors (Light loads - 1,5 to 2,5 kN/m^2)			

For shallower depths and heavy loads, deflection calculations may be required.

Materials

Introduction

Freshly mixed concrete is a combination of aggregates (inert material) and a paste of Portland cement and water. The aggregates used are generally sand and gravel or crushed stone. The aggregates have no cementing value of their own and serve purely as a filler. The final quality of the concrete depends on how effectively the hardened paste binds the aggregate together.

Desirable Properties of Fresh or Plastic Concrete.

The concrete must have the correct consistency or slump

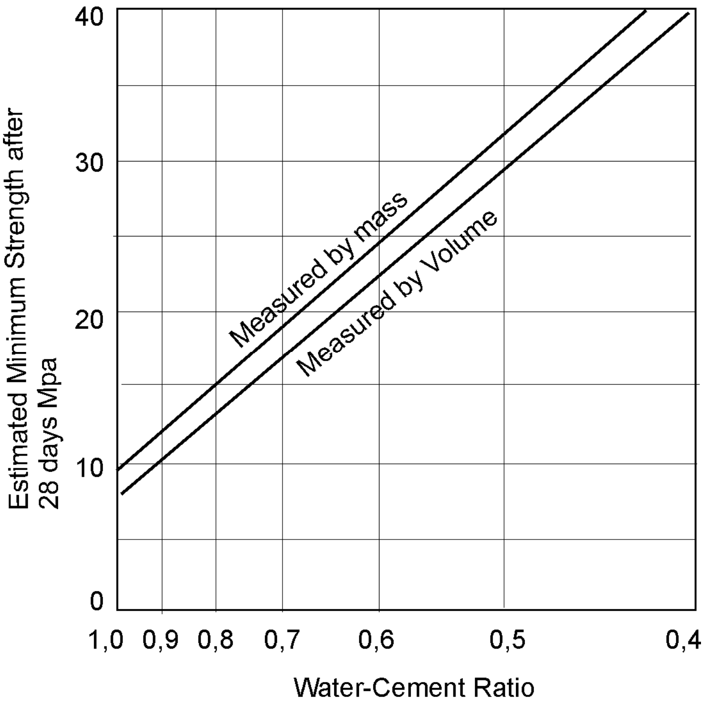
The concrete must be uniform, i.e., it must be mixed thoroughly
The concrete must be workable so that t can be placed and consolidated, removing all air pockets.

Desirable Properties of Hardened Concrete.

The concrete should be durable, strong, watertight and resistant to abrasion. All these properties are influenced by the quality of the cement paste and the aggregate.

Variable that Influence the Strength

Water-Cement Ratio



Aggregate Gradation

Bad gradation of aggregate

Good gradation of aggregate

Concrete properties such as relative proportions, workability, economy, porosity and shrinkage are affected by the grading and maximum size of the aggregate used.

Air Entrainment

The entrainment of air in concrete improves the resistance of the concrete to the adverse effects of freeze/thaw cycles. In addition, the ability of concrete to withstand the damaging effects of various deicing chemicals is improved. This resistance is particularly desirable in situations where the concrete is expected to be saturated with water.

Materials for the Production of Structural Concrete.

Introduction

Structural concrete is a combination of selected aggregate and a binding material obtained from a mixture of cement and water. Reinforcement in the form of round bars is placed in the concrete to improve the tensile properties of the concrete.

Cement

Portland cement is finely ground material consisting of compounds of lime, silica, alumina and iron. When mixed with water it forms a paste that hardens and binds the aggregate together.

Setting an Hardening

When cement is mixed with water a paste is formed which first sets and then hardens. The setting and hardening are due to the chemical reaction between the cement and water called hydration. If the concrete is kept moist, the hydration reaction will continue for years and the concrete will become progressively stronger and more durable.

Mixing Water for Concrete – How Impure can it be.

Generally if the water is clean enough to drink and has no pronounced taste or odour, it is clean enough to be used.

Aggregates for Concrete.

Sand, gravel and crushed stone are the aggregates most commonly used in concrete to provide volume at low cost. Since aggregates make up to about 60 to 80% of the volume of concrete, they can be called a filler material.
The aggregates should have the required strength and resistance to exposure conditions, i.e. abrasion, weathering etc. They should also not contain any dirt, clay, coal or organic material.

Reinforcement for Concrete

Round steel bars are generally used to reinforce concrete. The bars may be smooth or deformed to improve the bond between the steel and the concrete. Mild steel bars with a yield stress of 300 MPa are usually smooth whereas high tensile bars with a yield stress of 450 MPa are generally deformed.

Mild steel bars are denoted by having an R in front of the bar number and high tensile by a Y.

The following table gives standard sizes and cross-sectional areas of bars.

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.5	84.8	113.1	141.4	169.6	197.9	226.2	254.5	282.7
8	50.3	100.5	150.8	201.1	251.3	301.6	351.9	402.1	452.4	502.7
10	78.5	157.1	235.6	314.2	392.7	471.2	549.8	628.3	706.9	785.4
12	113.1	226.2	339.3	452.4	565.5	678.6	791.7	904.8	1017.9	1131.0
16	201.1	402.1	603.2	804.2	1005.3	1206.4	1407.4	1608.5	1809.6	2010.6
20	314.2	628.3	942.5	1256.6	1570.8	1885.0	2199.1	2513.3	2827.4	3141.6
25	490.9	981.7	1472.6	1963.5	2454.4	2945.2	3436.1	3927.0	4417.9	4908.7
32	804.2	1608.5	2412.7	3217.0	4021.2	4825.5	5629.7	6434.0	7238.2	8042.5

Application of Different Grades (Strengths) of Concrete

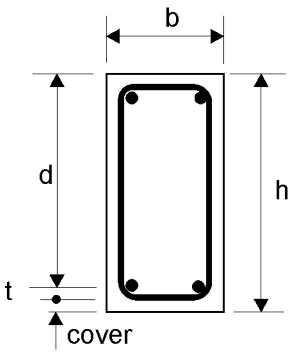
Grade	Lowest Grade for Use Specified
7 10	Plain Concrete of un-reinforced foundations and fill
15	Reinforced concrete with light-weight aggregate
20 25	Reinforced concrete with dense aggregate
30	Concrete with post-tensioned tendons
40 50 60	Concrete with pre-tensioned tendons

Table: Grades of Concrete and their uses.

Reinforcing Details

Concrete Cover

Concrete protects the steel reinforcement from corrosion by the environment. Greater cover may be required to protect the steel from fire.



	Nominal Cover (mm)				
Condition of exposure	20 MPa	25 MPa	30 MPa	40 MPa	50 or over
Mild: Completely protected against weather or aggressive conditions, except for a brief period of exposure	25	20	15	15	15
Moderate: Sheltered from severe rain and against freezing whist saturated with water. Buried concrete continuously under water	-	40	30	25	20
Severe: Exposed to driving rain, alternate wetting and drying and freezing whist wet. Subject to heavy condensation or corrosive fumes	-	50	40	30	25
Very Severe: Exposed to sea water or moorland water and abrasion	-	-	-	60	50
Subject to salt for de-icing	-	-	50*	40*	25

Nominal cover to reinforcement.

Reinforcement Spacing

Ensure that the spacing is such that the concrete flows easily between the bars. For example the minimum clear spacing between parallel bars on one layer and another should not be less than the bar diameter or 25 mm. The largest aggregate should not be greater than 75% of the clear space between bars.

Crack Control

Concrete has a low tensile strength and will be inclined to crack at low stress. Cracks are inevitable but their size must be restricted. Cracks reduce the effectiveness of concrete cover and decrease the stiffness and shear resistance of the member. Longitudinal reinforcement cannot eliminate cracks but will reduce the width while increasing the number giving small but closely spaced cracks.

Three types of cracks occur namely, *shrinkage cracks*, *temperature induced cracks* and *flexural cracks*.

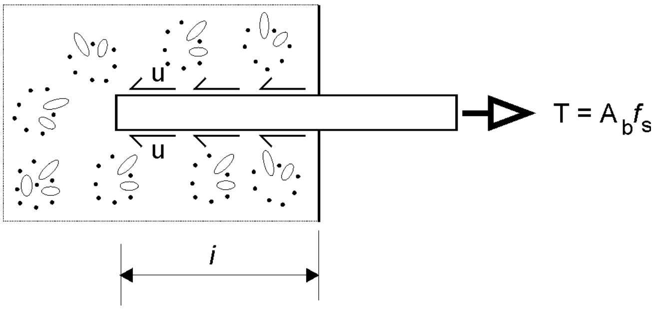
Shrinkage and temperature cracks
In one-way slabs reinforcement is placed vertical to the main reinforcement to prevent shrinkage and temperature cracks.

Flexural cracks

Since steel and concrete are bonded in a reinforced concrete structure, the deformations are the same. Crack widths due to the flexure are therefore a function of the stress in the steel and the spacing of the bars. To minimize the width of the flexural cracks, the stresses in the bars and their spacing must be as small as possible.

Anchorage and Development of Stress in Bars

We assume that all bars are bonded so that no slip occurs between the concrete and the steel. Between the steel and the concrete a stress will develop and this is called the bond stress.
Consider the horizontal equilibrium in the following sketch:



The bar has a cross-sectional area A_b and a stress of f_s is developed. This must be in equilibrium with the forces induced in the bond. The bond stresses are = u and the shear area is the circumference times the bond length *l*. One would like to ensure that the bond does not break before the steel bar yields. If the maximum bond stress is u_{max}, then:

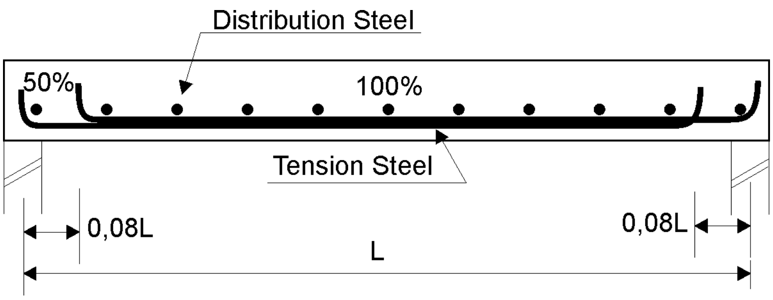
$$\pi \cdot d \cdot l \cdot u_{\max} \geq A_b \cdot f_y \geq \frac{\pi}{4} \cdot d^2 \cdot f_y$$

The required bond length is then: $l \geq \dfrac{d \cdot f_y}{4 \cdot u_{\max}}$

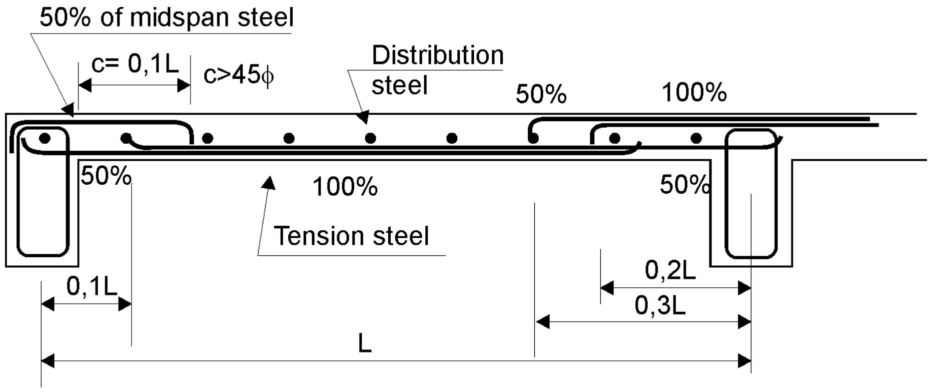
Arrangement of Reinforcement in Typical Concrete Members

Simple Beams and Slabs

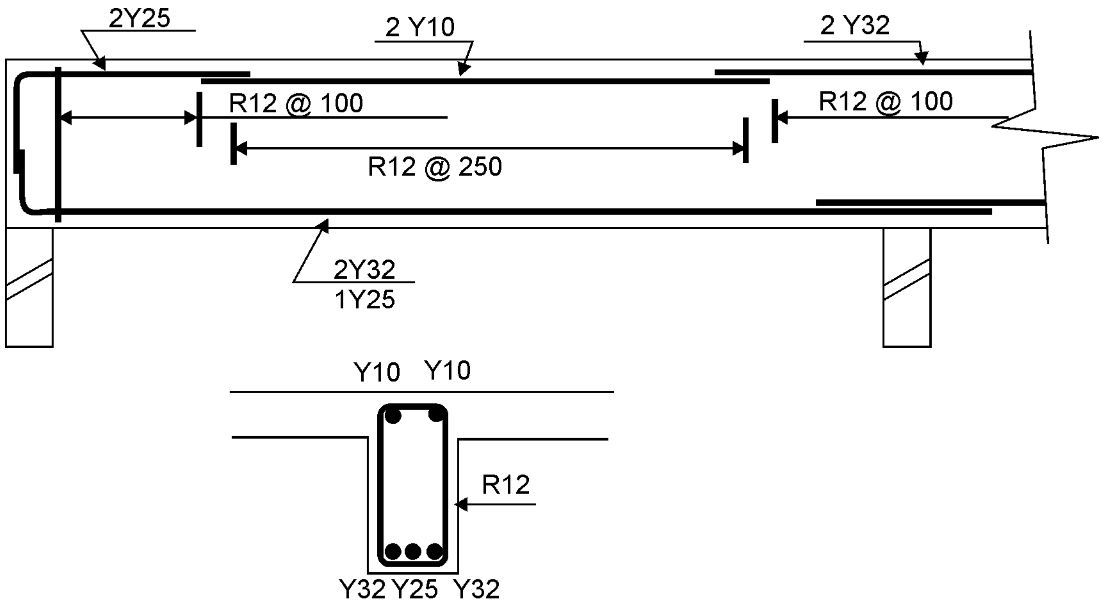
Simply supported slab:



Curtailment of tension steel in simply supported slab construction.



Curtailment of steel in beam and slab construction.



Typical steel detail for concrete beam.

Reinforced Concrete Design

Assumptions

Concrete has no tensile resistance

The reinforcing resists all the tensile forces.

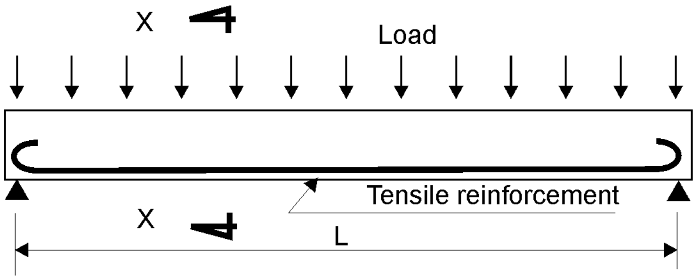
Maximum tensile stress in the steel: $f_{max} = \frac{f_y}{1{,}15} = 0{,}87 \cdot f_y$

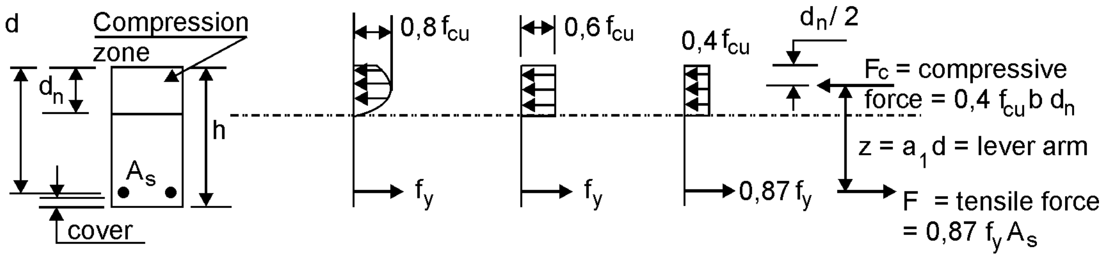
The characteristic cube stress of the concrete = f_{cu} and the compressive stress in the concrete = 0,4 f_{cu}

The maximum neutral axis depth (d_n) is limited to 0,5 d.

The internal moments are in equilibrium with the external moments.

Resistance moment (M_r) of a singly reinforced concrete beam or slab.





A partial material factor of 1,15 is applied to the steel and 1,5 to the concrete.

The following formula may be derived:

Resistance moment of the concrete in the compression zone:

Take moments about the tension zone, i.e., centroid of the tensile steel.

$$M_{rc} = \textit{compressive force x lever arm}$$
$$M_{rc} = F_c \cdot z$$
$$M_{rc} = 0{,}4 \cdot f_{cu} \cdot b \cdot d_n \cdot \left(d - \frac{d_n}{2} \right)$$

Assume a practical limit for the maximum: maximum d_n = 0,5 d

$$M_{rc(\max)} = 0{,}4 \cdot f_{cu} \cdot b \cdot (0{,}5 \cdot d) \cdot \left(d - \frac{0{,}5 \cdot d}{2} \right)$$
$$M_{rc(\max)} = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2$$

The resistance moment of the tensile zone:

Take moments about the centroid of the compression zone.

$$M_{rt} = \textit{tensile force x lever arm}$$
$$M_{rt} = F_t \cdot z$$
$$M_{rt} = 0{,}87 \cdot f_y \cdot A_s \cdot z$$

Neutral axis depth and lever arm with known beam section details:

Note that b, d and A_s are known.

Horizontal equilibrium of forces:

$F_c = F_t$

$$0,4 \cdot f_{cu} \cdot b \cdot d_n = 0,87 \cdot f_y \cdot A_s$$

$$d_n = \frac{0,87 \cdot f_y \cdot A_s}{0,4 \cdot f_{cu} \cdot b} = \textit{neutral axis depth}$$

The lever arm z

$$z = d - \frac{d_n}{2}$$

$$\textit{it follows that } z = d - \frac{0,87 \cdot f_y \cdot A_s}{0,8 \cdot f_{cu} \cdot b} = \textit{lever arm}$$

Determination of the lever arm z = $a_1$xd and tensile area A_s for a known section and known dimensions b and d with a moment M_u acting on the section.

$$d_n = n_1 \cdot d \textit{ and } z = a_1 \cdot d$$

$$z = d - \frac{d_n}{2} = d - \frac{n_1 \cdot d}{2} = \left(1 - \frac{n_1}{2}\right) \cdot d = a_1 \cdot d$$

$$n_1 = 2\left(1 - a_1\right) \therefore d_n = n_1 \cdot d$$

Substitute n_1 and z in the equation for M_u

$$M_u = 0,4 \cdot f_{cu} \cdot b \cdot (n_1 \cdot d)(a_1 \cdot d)$$

$$\frac{M_u}{f_{cu} \cdot b \cdot d^2} = 0,4 \cdot n_1 \cdot a_1 = 0,4(2)(1 - a_1)(a_1) = 0,8 \cdot a_1 - 0,8 \cdot a_1^2$$

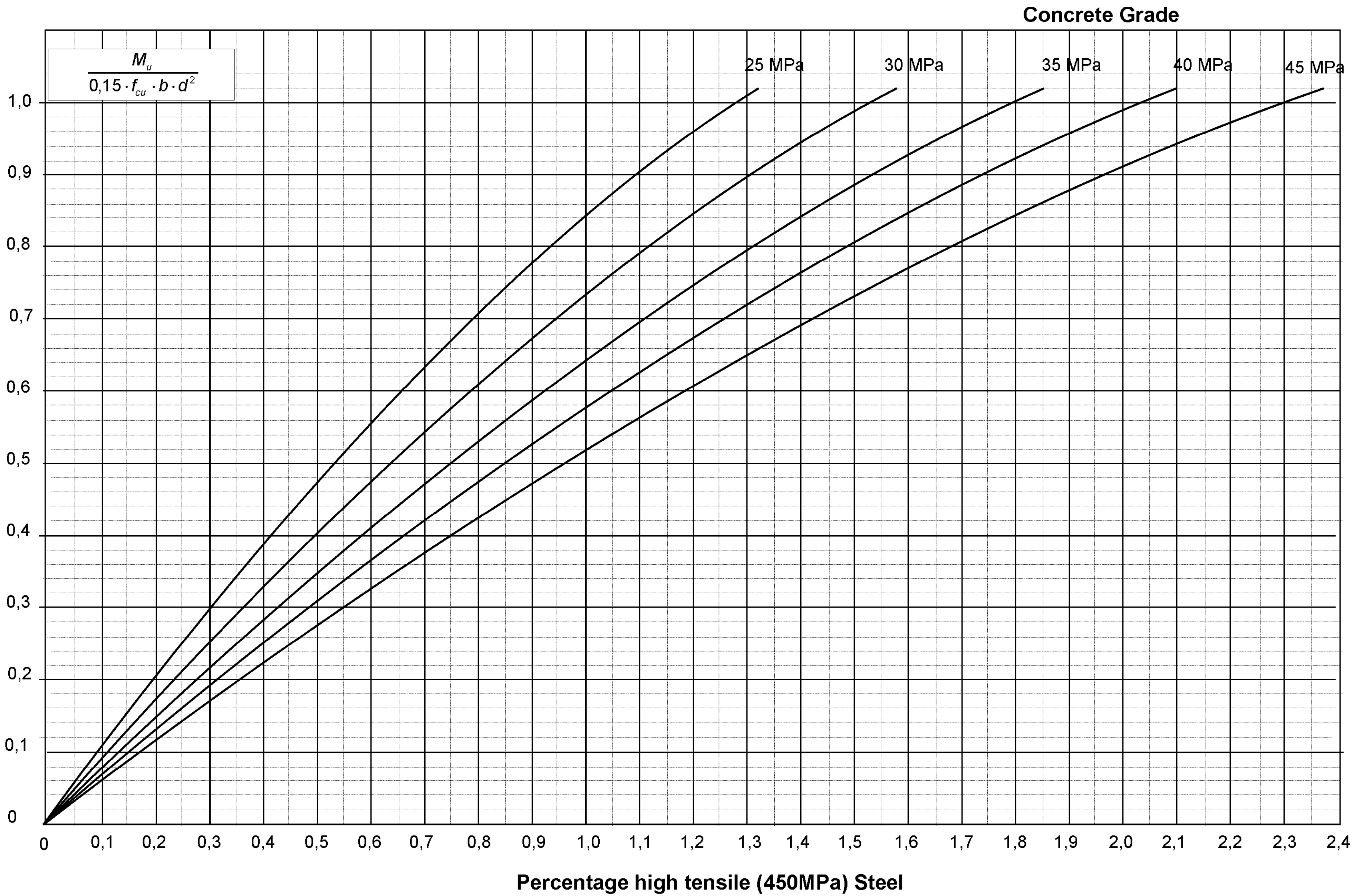
Solve for a_1:

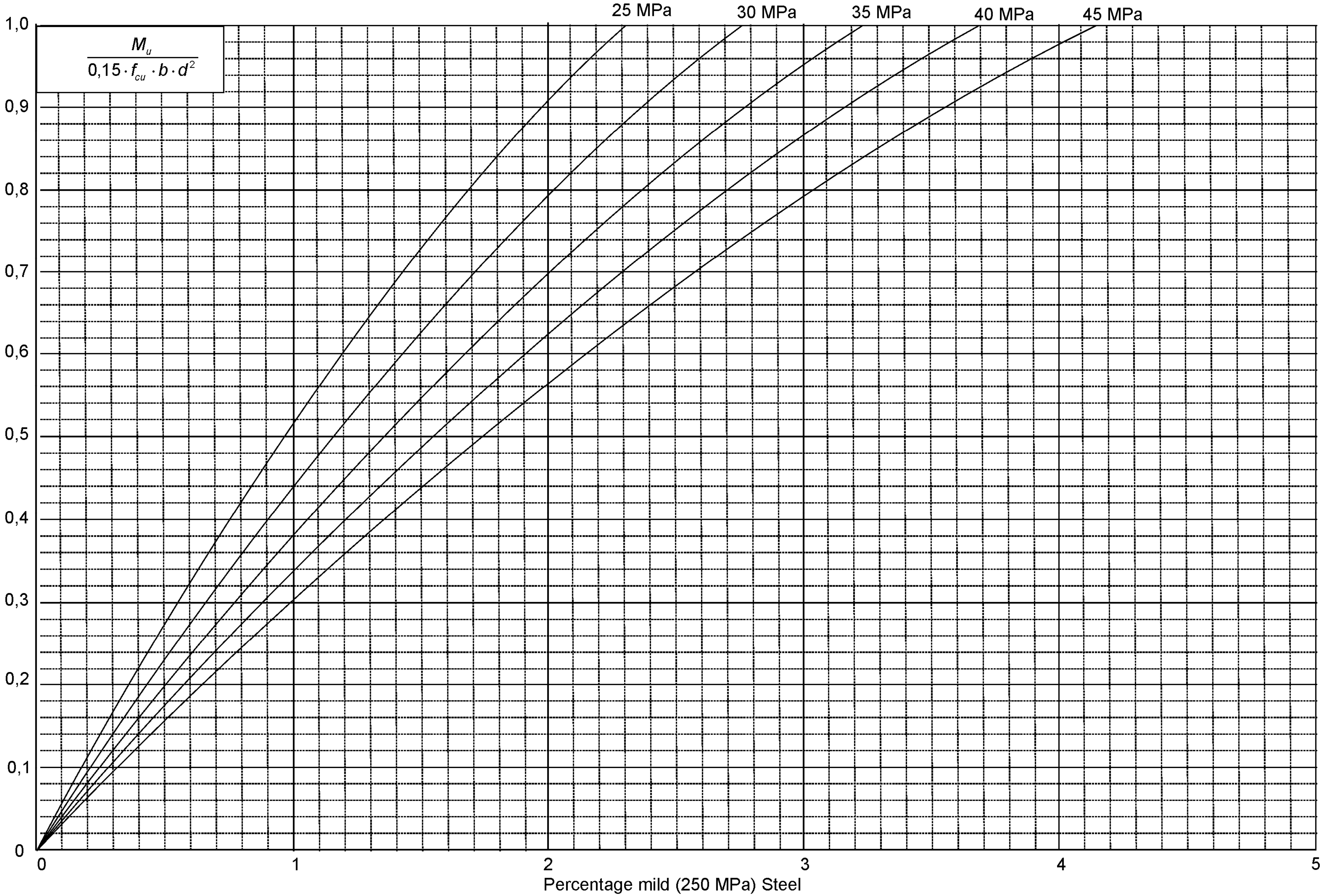
$$a_1 = \frac{1 + \sqrt{1 - \frac{5 \cdot M_u}{f_{cu} \cdot b \cdot d^2}}}{2}$$

This can be written in a tabular form:

$\frac{M_u}{f_{cu} \cdot b \cdot d^2}$	0,15	0,13	0,10	0,07	0,04
a_1	0,75	0,80	0,85	0,90	0,95

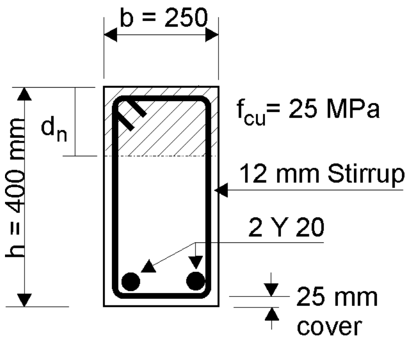
With a known value of a_1 the lever arm z may be calculated. Once the lever arm is know, the steel may be calculated.


Concrete Grade
$\frac{M_u}{0{,}15 \cdot f_{cu} \cdot b \cdot d^2}$
25 MPa
30 MPa
35 MPa
40 MPa
45 MPa
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0 0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0 1,1 1,2 1,3 1,4 1,5 1,6 1,7 1,8 1,9 2,0 2,1 2,2 2,3 2,4
Percentage high tensile (450MPa) Steel



25 MPa
30 MPa
35 MPa
40 MPa
45 MPa
$\frac{M_u}{0{,}15 \cdot f_{cu} \cdot b \cdot d^2}$
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
0
1
2
3
4
5
Percentage mild (250 MPa) Steel

Example 1:
Determine the neutral axis depth if the following beam dimensions are known. Also calculate n_1 from the equation $d_n = n_1$ d.



The neutral axis depth is given by:

$$d_n = \frac{0{,}87 \cdot f_y \cdot A_s}{0{,}4 \cdot f_{cu} \cdot b}$$

From tables of the areas of round bars, A_s = 628 mm^2

$$d_n = \frac{0{,}87 \cdot 450 \cdot 628}{0{,}4 \cdot 25 \cdot 250} = 98\ mm$$

$$d = h - cover - stirrup - \frac{bar\ diameter}{2}$$

$$\therefore d = 400 - 25 - 12 - \frac{20}{2} = 353\ mm$$

$$n_1 = \frac{d_n}{d} = \frac{98}{353} = 0{,}28 << 0{,}5\ the\ maximum$$

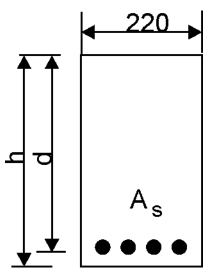
n_1 is small because the steel is very little.

$$\%\ steel = \frac{628}{250 \cdot 353} \cdot 100 = 0{,}71\%$$

From the graph it can be seen that M_u/M_r = 0,64 which tells us that the concrete in the beam still has additional capacity.

Example 2:
A 220 mm wide beam is simply supported and spans 6 m. The beam carries the following loads.
Dead load = 12 kN/m
Live load = 18 kN/m

Concrete stress, f_{cu} = 30 MPa
Reinforcing stress, f_y = 450 MPa



w_u = 1,2 x DL + 1,6 x LL

= 1,2 x (12) + 1,6 x (18)
= 43,2 kN/m

$$M_u = \frac{w_u \cdot L^2}{8} = \frac{43{,}2 \cdot 6^2}{8} = 194\ kN.m$$

$$M_{rc(\max)} = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2$$

Set the applied ultimate moment = the resistance moment

$$M_u = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2$$

Solve for d

$$d = \sqrt{\frac{M_u}{0{,}15 \cdot f_{cu} \cdot b}} = \sqrt{\frac{194x10^6}{0{,}15 \cdot 30 \cdot 220}} = 443\ mm$$

To this add cover and stirrup and round off to nearest 25 mm for practical reasons. Assume a 25 mm reinforcing bar. $h = 443 + 25 + 12 + \frac{25}{2} = 492{,}5$. Set h = 500 mm

Assume a bar diameter of 25 mm, stirrup = 12 mm and a cover of 25 mm. The new d = 500 - 25-12 - 25/2 = 450 mm.

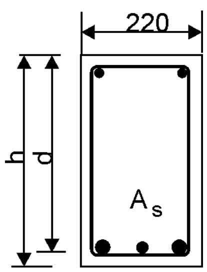
$$\frac{M_u}{0{,}15 \cdot f_{cu} \cdot b \cdot d^2} = \frac{194x10^6}{0{,}15 \cdot 30 \cdot 220 \cdot 450^2} = 0{,}97$$

From the graph we can read the percentage steel required:

% steel = 1,45%, i.e., $A_s = 0{,}0145 \cdot 220 \cdot 450 = 1436\ mm^2$

Bar size (mm)	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.5	84.8	113.1	141.4	169.6	197.9	226.2	254.5	282.7
8	50.3	100.5	150.8	201.1	251.3	301.6	351.9	402.1	452.4	502.7
10	78.5	157.1	235.6	314.2	392.7	471.2	549.8	628.3	706.9	785.4
12	113.1	226.2	339.3	452.4	565.5	678.6	791.7	904.8	1017.9	1131.0
16	201.1	402.1	603.2	804.2	1005.3	1206.4	1407.4	1608.5	1809.6	2010.6
20	314.2	628.3	942.5	1256.6	1570.8	1885.0	2199.1	2513.3	2827.4	3141.6
25	490.9	981.7	1472.6	1963.5	2454.4	2945.2	3436.1	3927.0	4417.9	4908.7
32	804.2	1608.5	2412.7	3217.0	4021.2	4825.5	5629.7	6434.0	7238.2	8042.5

Steel required = 3 ϕ 25 (1470)



Example 3:

A slab is placed over a double garage with dimensions 6 m x 10 m. The space above is to be used as a studio with office loading of 2,5 kN/m^2. The floor finish has a weight of 1,0 kN/m^2. Determine the slab thickness, the weight on the slab and the reinforcing. Assume that the slab is simply supported on all 4 sides. Use a 25 MPa concrete, high tensile reinforcing and 20 mm cover.

Slab thickness = smaller span / 28 = 6000 / 28 = 214 mm
Use standard brick plus mortar layer dimensions of 85 mm, therefore use 3 x 85 = 225 mm.

Self weight of slab	= 0,225 x 24 kN/m^3	= 5,4 kN/m^2
Floor finish		= 1,0 kN/m^2
Total permanent or dead load		= 6,4 kN/m^2
Live load		= 2,5 kN/m^2
Factored load w_u	= 1,2 x DL + 1,6 x LL	= 1,2 x 6,4 + 1,6 x 2,5
		= 11,68 kN/m^2

Assume a 1 m wide strip of the slab.

The uniformly distributed load on the 1 m wide slab is then 11,68 kN/m

Ratio of longer span to shorter span = $\frac{L_y}{L_x} = \frac{10}{6} = 1{,}67$

Ly/Lx	1,0	1,1	1,2	1,3	1,4	1,5	1,75	2,0	2,5	3,0
α_X	0,062	0,074	0,084	0,093	0,099	0,104	0,133	0,118	0,122	0,124
α_Y	0,062	0,061	0,059	0,055	0,051	0,046	0,037	0,029	0,020	0,014

Table for moments in the two directions:
α_{SX} = 0,110
α_{SY} = 0,040

$$M_{ux} = \alpha_{sx} \cdot w_u \cdot L_x^2 = 0{,}110 \cdot 11{,}68 \cdot 6^2 = 46{,}25\ kN.m$$

$$M_{uy} = \alpha_{sy} \cdot w_u \cdot L_x^2 = 0{,}040 \cdot 11{,}68 \cdot 6^2 = 16{,}82\ kN.m$$

Assume 16 mm diameter bars. Depth to the centre of the reinforcing bars = 255 – 20 – 16/2 = 227 mm

$$\frac{M_{ux}}{0{,}15 \cdot f_{cu} \cdot b \cdot d^2} = \frac{46{,}25x10^6}{0{,}15 \cdot 25 \cdot 1000 \cdot 227^2} = 0{,}24$$

$$\frac{M_{uy}}{0{,}15 \cdot f_{cu} \cdot b \cdot d^2} = \frac{16{,}82x10^6}{0{,}15 \cdot 25 \cdot 1000 \cdot (227-16)^2} = 0{,}101$$

From the graph we can determine the percentage steel:

% A_{SX} = about 0,24%
% A_{SY} = about 0,12%

Use a minimum of 0,12% for the steel in the Y-direction.

A_{SX} = 0,24 x 1000 x 227 /100 = 545 mm^2
A_{SY} = 0,12 x 1000 x 211 /100 = 254 mm^2

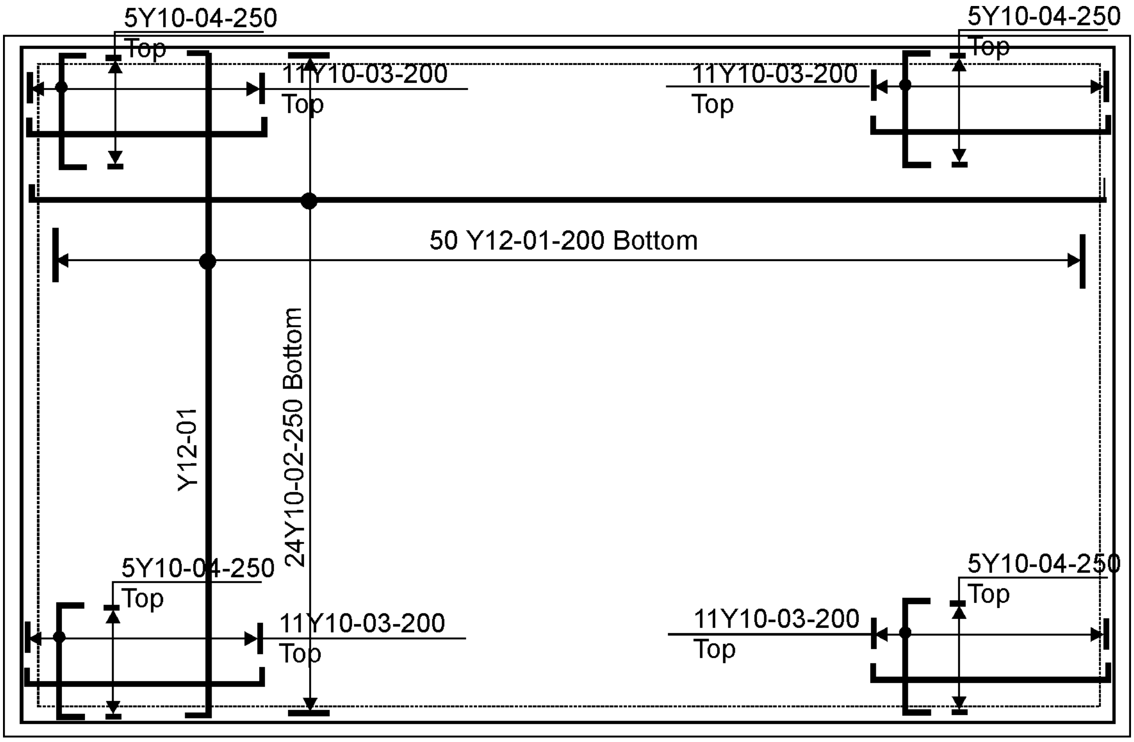
Spacing (mm)	Bar Diameter						
	8	10	12	16	20	25	32
100	503	785	1131	2011	3142	4909	8042
125	402	628	905	1608	2513	3927	6434
150	335	524	754	1340	2094	3272	5362
200	251	393	565	1005	1571	2454	4021
225	223	349	503	894	1396	2182	3574
250	201	314	452	804	1257	1963	3217
275	183	286	411	731	1142	1785	2925
300	168	262	377	670	1047	1636	2681

Cross-section area of reinforcing bars spaced at various intervals.

Steel in the shorter direction use ϕ 12 mm at 200 mm spacing with an area of 565 mm^2.
Steel in the shorter direction use ϕ 10 mm at 250 mm spacing with an area of 314 mm^2.

It is always a good idea to supply torsional steel in the corners of slabs as these will tend to crack if not reinforced properly. It should consist of top and bottom steel each with layers of bars placed parallel to the sides of the slab and extending a distance of one fifth of the span. The area should be three quarters of the maximum steel in the centre.

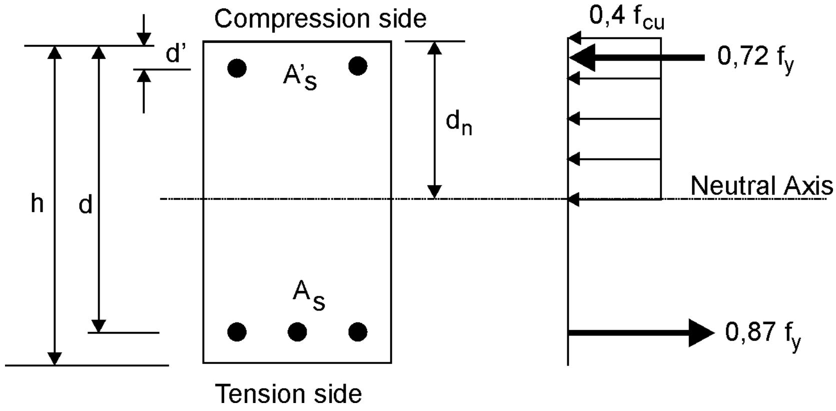
75% of 545 = 409 mm^2. Use Y10 @200 with an area of 393 mm^2.
Use minimum steel in Y-direction.



Typical layout for a simply supported flat slab.

Doubly reinforced beams.

If the ultimate moment is greater than the maximum resistance moment of the beam, i.e., Mu > Mrc (Concrete) Then additional steel may be provided on the compression side of the member. The strain at which this steel functions is limited by the strain at which the concrete functions at ultimate strength. The resistance of the steel is then limited by the strain.



Section with compression and tension steel.

The total resistance moment of the section, $M_r = M_{rc(max)} + M'_{rs}$

$M'_{sc} = f_{sc} \cdot A'_s \cdot (d - d')$ Compression steel resistance moment

$M'_{sc} = 0{,}72 \cdot f_y \cdot A'_s \cdot (d - d')$

$M_{rc(\max)} = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2$ Concrete resistance moment

It is now possible to determine the area of the compression steel.

$$A'_s = \frac{M_u - 0{,}15 \cdot f_{cu} \cdot b \cdot d^2}{0{,}72 \cdot f_y \cdot (d - d')}$$

For horizontal equilibrium:

Compression force	=	Tension force
$\frac{0{,}4 \cdot f_{cu} \cdot b \cdot d}{2} + 0{,}72 \cdot f_y \cdot A'_s$	=	$0{,}87 \cdot f_y \cdot A_s$

It then follows that the tensile steel area:

$$A_s = \frac{0{,}23 \cdot f_{cu} \cdot b \cdot d}{f_y} + 0{,}82 \cdot A'_s$$

The total moment of resistance:

$$M_{rc(total)} = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2 + 0{,}72 \cdot f_y \cdot A'_s \cdot (d - d')$$

Example 4:

A 220 x 500 x 25 MPa beam has an applied ultimate moment of 200 kN.m. Determine the required reinforcing in the member.

The ultimate maximum resistance moment of the concrete. Estimate the bar size to be 2 layers of 25 mm with a 25 mm between. The cover is 25 mm. The depth d = 500 –25 (cover) – 12 (stirrup) – 25 (bar size) – 25/2 (half distance between bars) = 425 mm

$M_{rc(\max)} = 0{,}15 \cdot f_{cu} \cdot b \cdot d^2 = 0{,}15 \cdot 25 \cdot 220 \cdot 425^2 \cdot 10^{-6} = 149\ kN.m$. The beam requires compression steel.

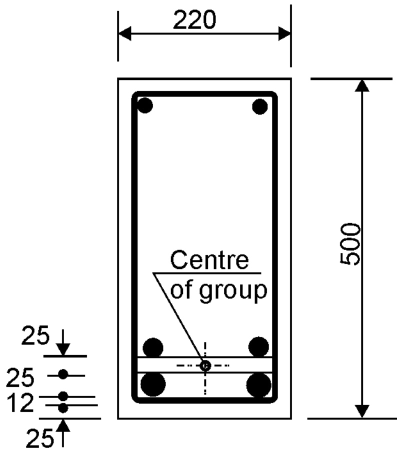
Assume 25 mm bars in compression with 25 mm cover to the steel. The distance to the centre of the compression steel, d', = 25 + 12 (stirrup) + 25/2 = 50 mm

$$A'_s = \frac{M_u - 0{,}15 \cdot f_{cu} \cdot b \cdot d^2}{0{,}72 \cdot f_y \cdot (d - d')} = \frac{(200 - 149) \cdot 10^6}{0{,}72 \cdot 450 \cdot (425 - 50)} = 420\ mm^2$$ (2 ϕ 20 mm bars with area of 628 mm^2)

Determine the tension reinforcement:

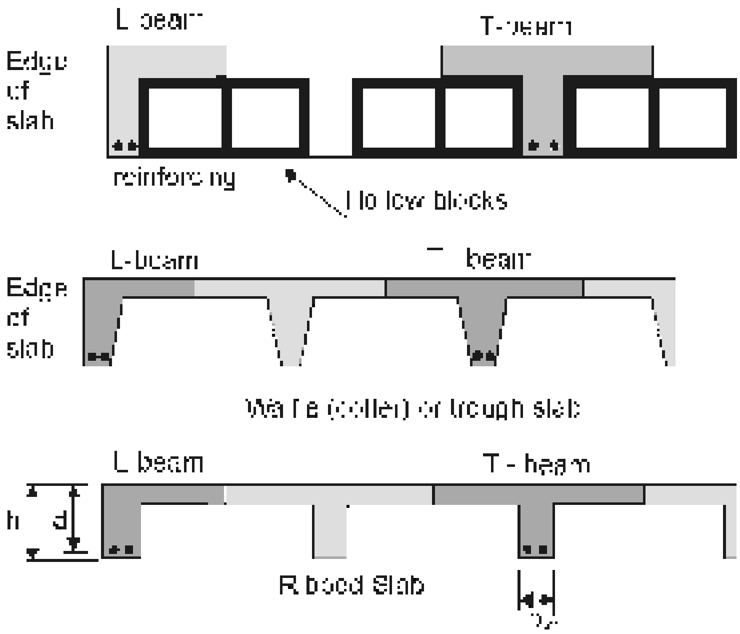
$$A_s = \frac{0{,}23 \cdot f_{cu} \cdot b \cdot d}{f_y} + 0{,}82 \cdot A'_s = \frac{0{,}23 \cdot 25 \cdot 220 \cdot 425}{450} + 0{,}82 \cdot 420 = 1539\ mm^2$$

Use 2 ϕ 25 (982 mm^2)+ 2 ϕ 20 (628 mm^2)



T and L – beams

T and L – beams are found when concrete is saved by using hollow block floors, waffle (coffer) slabs, slabs with ribs or trough slabs.



Thickness of slabs over hollow blocks and between ribs.

The following may be used as a rough estimate:

Hollow block floors: 40 mm or one-tenth of the distance between ribs, whichever is greater.
Ribbed floors without permanent blocks: 50 mm or one-tenth of the distance between ribs, whichever is greater.

Rib-width should not be less than 65 mm and spaced at no more than 1500 mm.

Effective width of compression flange.

The effective width of the compression flange depends on the spacing of the ribs and the span and is not necessarily the distance between the ribs. The effectiveness is determined by the shear flow.

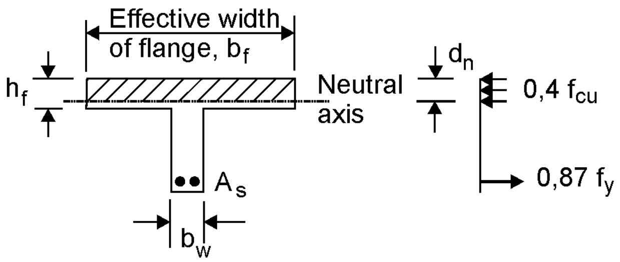
T-beam: The lesser of (a) the width of the web b_w plus one-fifth of the distance between positions of zero moment or (b) the spacing of the ribs.

L-beam: The lesser of (a) the width of the web b_w plus one-tenth of the distance between positions of zero moment or (b) the spacing of the ribs.

Design Procedure:

Two cases are possible:

Firstly the neutral axis may lie in the slab.



Assume that the neutral axis is in the flange:

Calculate the lever arm, $z = a_1$ d.

$$a_1 = \frac{1 + \sqrt{1 - \frac{5 \cdot M_u}{f_{cu} \cdot b \cdot d^2}}}{2}$$

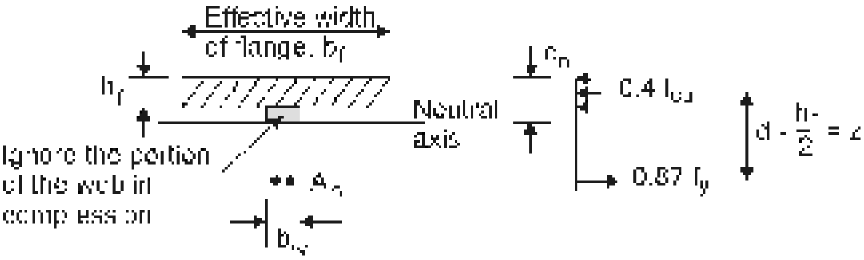
With the value of z known it is possible to calculate d_n using the equation:

$$z = d - \frac{d_n}{2}$$
$$\therefore d_n = 2 \cdot (d - z)$$

If d_n is less than h_f calculate the tension steel as usual:

$$A_s = \frac{M_u}{0{,}87 \cdot f_y \cdot z}$$

Secondly the neutral axis may lie in the web.



This will happen if d_n is greater than h_f.

The resistance moment of the concrete is given by:

$M_{rc} = 0{,}4 \cdot f_{cu} \cdot b_f \cdot h_f \cdot \left(d - \frac{h_f}{2} \right)$ and this must be greater than the applied ultimate moment, M_u. If M_u is greater than the resistance moment of the concrete, compression steel must be added.

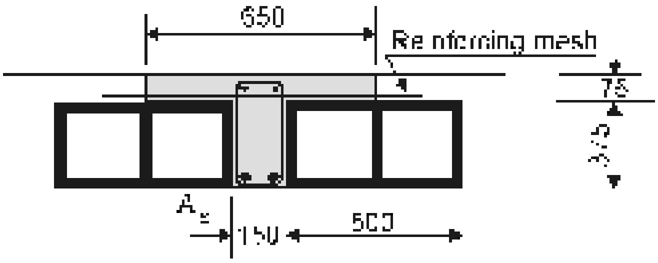
If M_{rc} > than M_u:

$$A_s = \frac{M_u}{0{,}87 \cdot f_y \left(d - \frac{h_f}{2} \right)}$$

Example 5:

A hollow block floor spans 8 m. The blocks are 500 x 375 mm and weigh 1 kN/m. The floor finish has a weight of 0,9 kN/m^2. A live load of 2,5 kN/m^2 is applied to the floor. Determine the required reinforcing to the beam. Concrete 25 MPa after 28 days.

Preliminary size



In a 650 mm wide strip:

Dead load hollow block:			= 1,00 kN/m
Concrete	Area x unit weight	= (0,65 x 0,075 + 0,375 x 0,15) x 24	= 2,52 kN/m
Floor finish	width x unit weight	= 0,65 x 0,9	= 0,586 kN/m
Total dead load			**= 4,106 kN/m**

Live load width x unit load = 0,65 x 2,5 = 1,625 kN/m

Factored load 1,2 x DL + 1,6 x LL = 1,2 x 4,106 + 1,6 x 1,625 = 7, 53 kN/m

Ultimate moment $M_u = \frac{w_u \cdot L^2}{8} = \frac{7{,}53 \cdot 8^2}{8} = 60{,}24\ kN.m$

Cover = 25 mm. Assume 25 mm high tensile bars.
d = 450 – 25 –10 (stirrups)– 25/2 = 402,5 mm, lets call it 400 mm

Calculate the lever arm

$$a_1 = \frac{1+\sqrt{1-\frac{5\cdot M_u}{f_{cu}\cdot b\cdot d^2}}}{2} = \frac{1+\sqrt{1-\frac{5\cdot 60,24\cdot 10^6}{25\cdot 650\cdot 400^2}}}{2} = 0,97 \qquad \text{Maximum value} = 0,95$$

z = a_1. d 0,95 . 400 = 380 mm

$$z = d - \frac{d_n}{2}$$

$$\therefore d_n = 2\cdot(d-z) = 2\cdot(400-380) = 40\ mm$$

The neutral axis falls within the flange.

$$A_s = \frac{M_u}{0,87\cdot f_y\cdot z} = \frac{60,24\cdot 10^6}{0,87\cdot 450\cdot 380} = 405\ mm^2$$

Use 2 x ϕ 16 mm bars with an area of 402 mm^2.

Cover = 25 mm. Assume 16 mm high tensile bars.
d = 450 – 25 –10 (stirrups)– 16/2 = 407 mm
z = a_1. d 0,95 . 407 = 386 mm

$$A_s = \frac{M_u}{0,87\cdot f_y\cdot z} = \frac{60,24\cdot 10^6}{0,87\cdot 450\cdot 386} = 399\ mm^2$$

Alternative design method:

Use the design table. $\frac{M_{ux}}{0,15\cdot f_{cu}\cdot b\cdot d^2} = \frac{60,24x10^6}{0,15\cdot 25\cdot 650\cdot 407^2} = 0,15$

% steel required = 0,15%

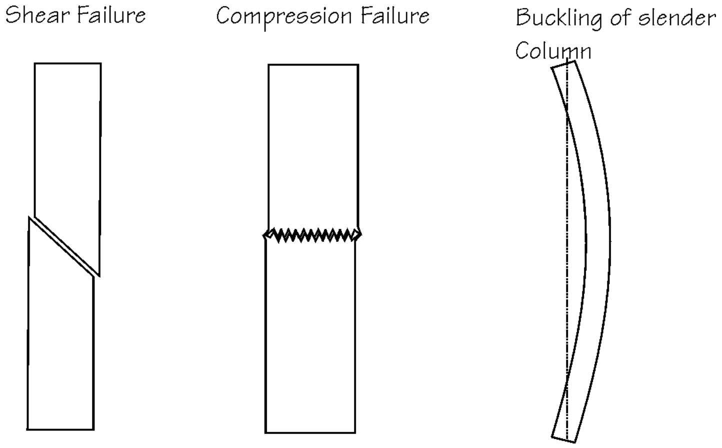
A_s = 0,15 x 650 x 407/100 = 397 mm^2

Reinforced Concrete Columns

Columns carry the loads from roofs, floors and beams to the foundations (footings) in compression and bending. Bending moments may be present if frame action is used to carry wind loads and beam loads. In this course only axial loads will be taken into account.

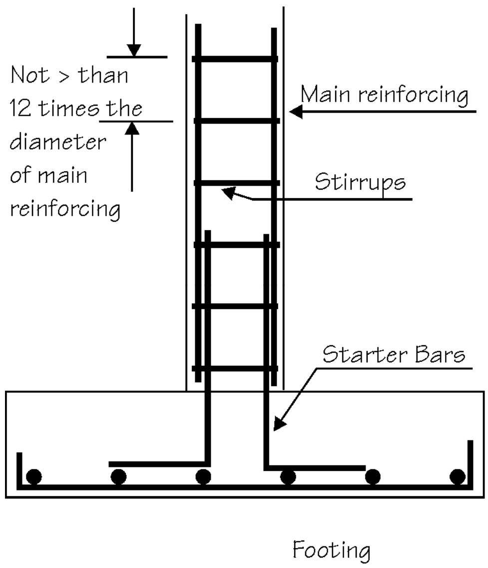
Failure of columns.

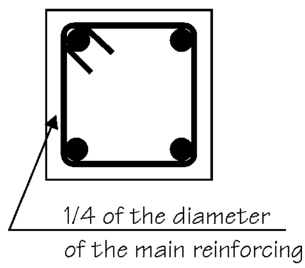
Columns may fail in one of or a combination of the following failure mechanisms.



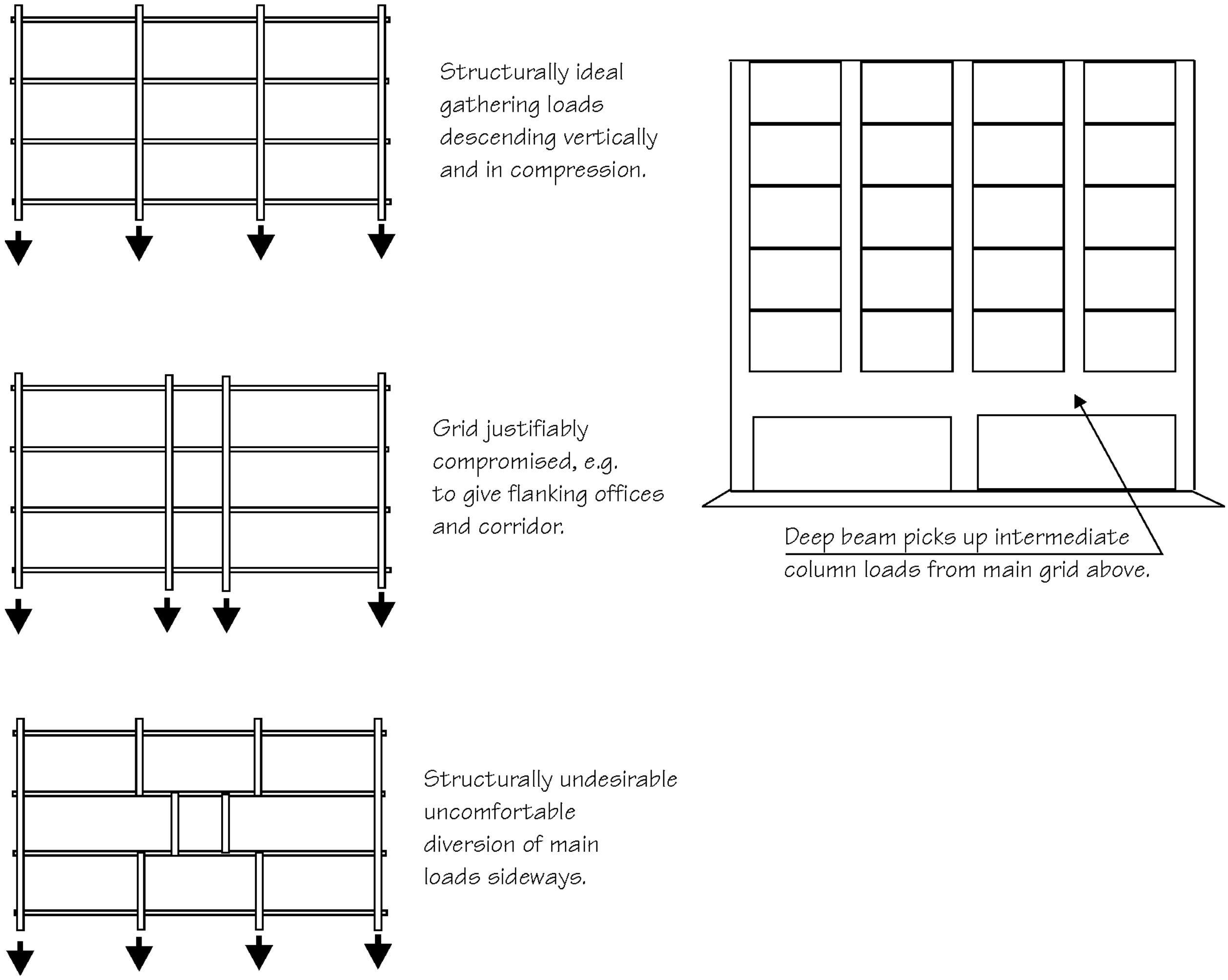
Reinforcing of Concrete Columns

Reinforcing increases the capacity of the column to resist both axial forces and bending moments.





Column Alignment and Spacing

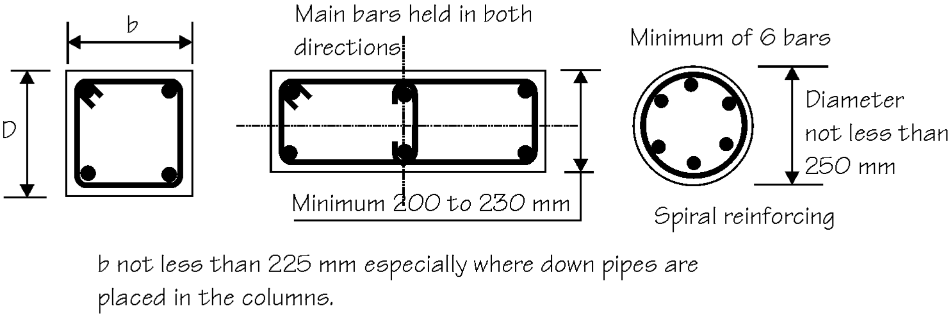


Reducing the spacing of columns will lead to reduced slab and beam thicknesses. Too many columns close together will limit the floor space and the structure will appear to be “busy”. For planning, a minimum column spacing of 4 m in the one direction and 5 m in the other direction is recommended with an ideal spacing of between 6 m and 7,5 m in both directions. In some situations such as the basement of high buildings, a column spacing of 10 m may be both functional and economical.

Beam and slab construction, coffer slabs and post-tensioned slabs make bigger column spacing possible, i.e., 10 m to 12 m.

Column Shapes and Economy

It is possible to cast concrete columns in almost any cross-sectional shape. Unusual shapes require made-for-purpose shuttering, which will be expensive. Symmetrical shapes such as rectangles and round are the preferred shapes.



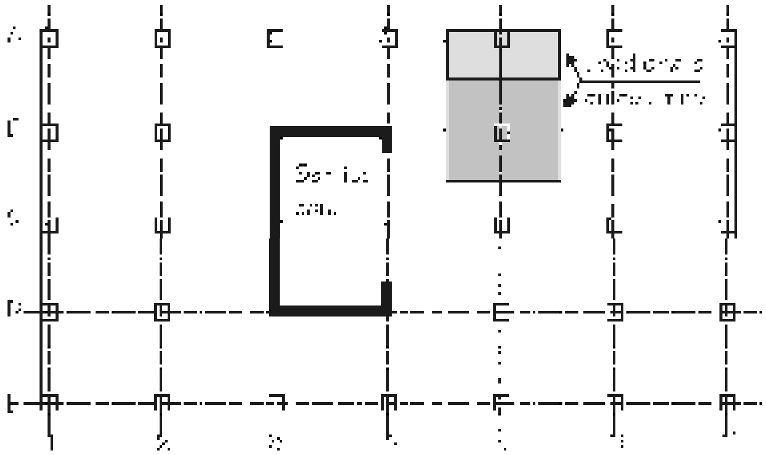
It is uneconomical to change the column dimensions on each floor of multi-storey buildings just to reflect the change in the load. The dimensions are usually kept constant and the amount of reinforcing is varied between 1% and 6%.

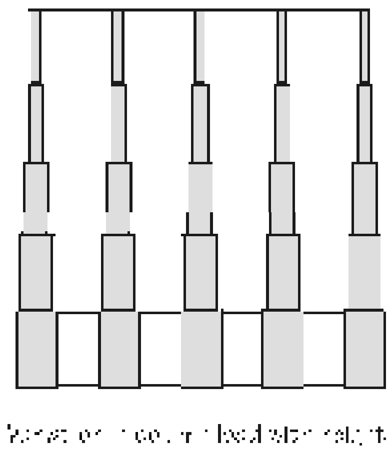
Shutter suppliers generally keep standard shutter sizes. Deviation from standard sizes can increase the cost of the building. The following sizes should be considered:

Typical column boxes.

Cross-sectional dimensions: 225 mm to 900 mm in increments of 75 mm.
Column height: up to 3,6 m.

Vertical Loads Carried by Columns.

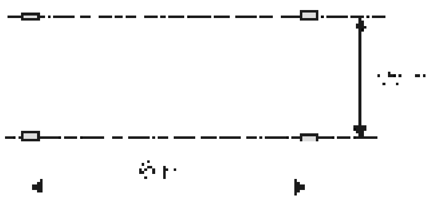




Column load varies not only in plan but also in height.

Practical Estimated Column Sizes for Multi-Storey Buildings.

Keep minimum parking bay size in mind if there is basement or other parking:



Typical minimum parking bay size.

The following table may be used to estimate the size of concrete columns in multi-storey office or apartment buildings.

Column spacing (mxm)	For the bottom level: Estimated column sizes (mm x mm)					
	Number of storeys – offices or apartments					
	2	4	6	8	10	12
5 x 5				400 x 400		
6 x 6			400 x 400	500 x 500		
7 x 7	300 x 300	375 x 375	450 x 450	600 x 600		

Design of Columns for Adequate Axial Strength.

Short stocky columns usually fail in crushing of the material whereas long slender columns may buckle. To avoid buckling ensure that the slenderness is not too large>

Stocky columns $\frac{l_e}{b_{\min imum}} \leq 12$

Slender columns $\frac{l_e}{b_{\min imum}} > 12$

l_e is the effective length
b_{min} is smaller cross-sectional dimension.

Slender columns require special design considerations, this course will be restricted to stocky columns.

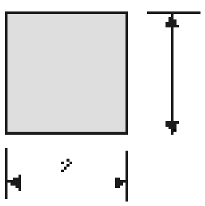
Axial Resistance of Stocky Columns.

The axial resistance: $C_r = 0{,}35 \cdot f_{cu} \cdot A_c + 0{,}60 \cdot f_y \cdot A_{sc} = C_{r(concrete)} + C_{r(steel)}$

f_{cu} = characteristic concrete cube strength
fy = yield stress of the reinforcing
A_c = cross-sectional area of the concrete
A_{sc} = cross-sectional area of the reinforcing steel.

The resistance, C_r must be greater than the ultimate load, C_u.
C_u is usually based on 1,2 x DL + 1,6 x LL.

For initial design purposes it is recommended that A_{sc} < 2% of the concrete cross-sectional area.



A_{sc} < 0,02 A_g where A_g = b x b.
Because $A_g = A_c + A_{sc}$
It follows that A_c = 0,98 A_g
Then $C_r = 0{,}35 \cdot f_{cu} \cdot 0{,}98 \cdot A_g + 0{,}60 \cdot f_y \cdot 0{,}02 \cdot A_g \geq C_u$

If C_u, f_{cu} and f_y are known it is easy to calculate $A_g = b^2$

Example 1:
A reinforced concrete column is 5 m long and carries the service loads of:
Dead load = 600 kN
Live Load = 400 kN.
Assume f_{cu} = 35 MPa and f_y = 450 MPa.

The column is fixed top and bottom. Determine a suitable column size and reinforcing.

Ensure that the column is stocky, i.e., $L_e/b < 12$.

L_e = 0,7 x L = 0,7 x 5000 = 3500 mm.

$$\frac{L_e}{b} \leq 12 \therefore b \geq \frac{L_e}{12} \geq \frac{3500}{12} \geq 292\ mm$$

Use a 300 x 300 mm column.

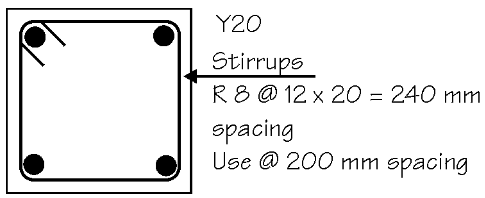
C_u = 1,2 x DL + 1,6 x Imposed Load = 1,2 x 600 + 1,6 x 400 = 1360 kN

$$C_r = 0{,}35 \cdot f_{cu} \cdot A_c + 0{,}6 \cdot f_y \cdot A_{sc} \geq C_u$$

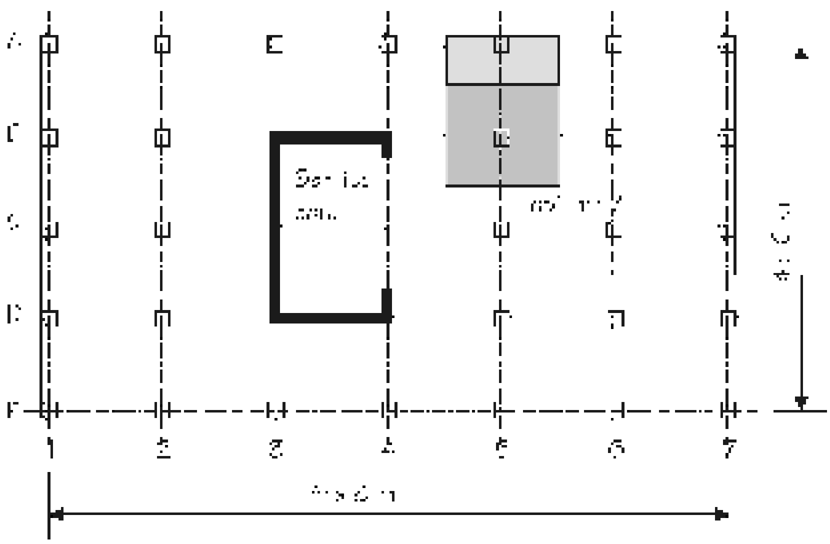
$$0{,}35 \cdot 35 \cdot (300x300 - A_{sc}) + 0{,}6 \cdot 450 \cdot A_{sc} \geq 1360x10^3$$

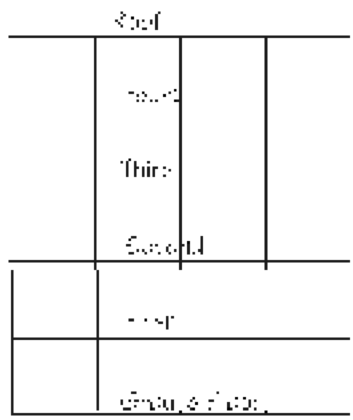
$1102500 - 12{,}25\ A_{sc} + 270\ A_{sc} > 1360 \times 10^3$
$A_{sc} > 999\ mm^2$

Use 4 Y20 with an area of 1256 mm^2.



Example 2:
Determine preliminary sizes of the columns of the 5 storey office block. Assume:
2,5 kN/m^2 for live load
0,5 kN/m^2 for partition loads
Concrete strength = 35 MPa
Steel yield strength = 450 MPa





Estimate the floor thickness = $\frac{Short\ span}{25} = \frac{6000}{25} = 240\ mm$

Use a thickness of 250 mm

Area carried by column = 8 x 6 m^2 / floor = 48 m^2

Total area carried by column	= 48 x 5 floors (first floor to roof inclusive) = 240 m^2

Dead or permanent load, DL:	Slab	= 0,25 x 24	= 6,0 kN/m^2
	Partitions		= 0,5 kN/m^2
	Total		= 6,5 kN/m^2

Imposed load (live load), IL		= 2,5 kN/m^2
w_u = 1,2 x DL + 1,6 x IL	= 1,2 x 6,5 + 1,6 x 2,5	= 11,8 kN/m^2
Total load on column	= Area x w_u = 240 x 11,8	 = 2832 kN

Assume that we are going to use 2% steel

$A_g = A_{concrete} + A_{sc}$

$$C_r = 0{,}35 \cdot f_{cu} \cdot A_c + 0{,}6 \cdot f_y \cdot A_{sc} \geq C_u$$

$$C_r = 0{,}35 \cdot f_{cu} \cdot 0{,}98 A_g + 0{,}6 \cdot f_y \cdot 0{,}02 A_g \geq 2832\ x\ 10^3$$

$$0{,}35 \cdot 35 \cdot 0{,}98 A_g + 0{,}6 \cdot 450 \cdot 0{,}02 A_g \geq 2832 x 10^3$$

$A_g > 162711$ mm^2

$b^2 > 162711$ mm^2

b > 403 mm

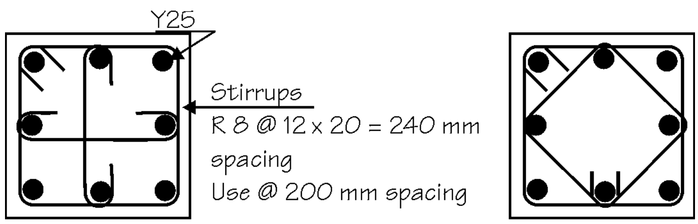
Use a 400 x 400 mm cross-section.

$$0{,}35 \cdot 35 \cdot (400 x 400 - A_{sc}) + 0{,}6 \cdot 450 \cdot A_{sc} \geq 2832 x 10^3$$

1969 x 10^3 – 12,25 A_{sc} + 270 A_{sc} > 2832 x 10^3

$A_{sc} > 3384$ mm^2

Use 8 Y 25 reinforcing rods.



Alternative

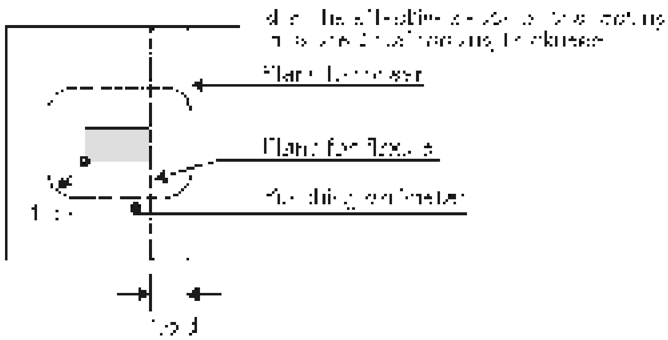
Concrete Footings

The primary purpose of a footing is to ensure that the axial loads from columns and walls are safely transferred to the ground and that the stress on the ground does not cause undue displacement. To ensure that the displacement is not too large, the stress as a result of service loading may not exceed the allowable. For the design of the reinforcing in the footing, ultimate loads must be used.

The following structural aspects must be investigated in the design of the footing.

i) The plan surface area must be large enough to safely transfer the service loads
ii) The footing must have sufficient resistance moment to resist the ultimate moment induced by the soil pressure
iii) The footing must have sufficient shear resistance to resist the ultimate shear induced by the soil pressure.
iv) The punching resistance must be large enough.

Foundations should be thick enough that punching shear and normal shear are not a problem.



Example 3:

Design a reinforced footing for the following:

- 300 x 300 mm column
- service load Dead load = 600 kN, Imposed load = 400 kN
- Allowable soil pressure = 300 kN/m^2 (kPa)
- f_{cu} = 25 MPa,
- f_y = 250 MPa

Solution:

Area of footing $A = \frac{Service\ load}{Allowable\ pressure} = \frac{600+400}{300} = 3{,}333\ m^2$

Use a square footing, $A = b^2 \ \therefore\ b = \sqrt{3{,}333} = 1{,}825\ mm$

Set b = 1850 mm

Assume a preliminary thickness of : $t = \frac{b}{4} = \frac{1850}{4} = 463\ mm$

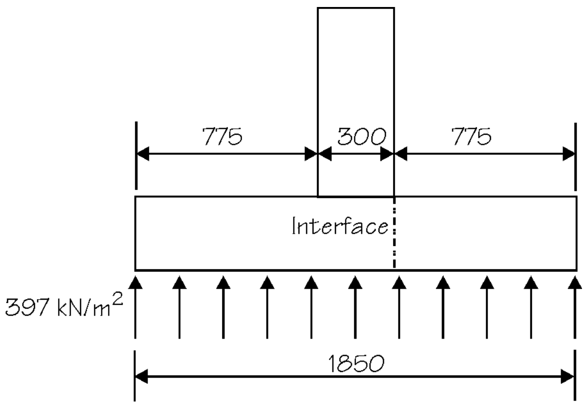
Use a thickness of 450 mm.

Design the footing for bending moment; we must therefore use ultimate forces and moments.

Ultimate load = 1,2 x DL + 1,6 x IL = 1,2 x 600 + 1,6 x 400 = 1360 kN

Pressure under the footing $= \frac{Force}{Area} = \frac{1360}{1{,}850x1{,}850} = 397\ kN/m^2 = 397\ kPa$

Calculate the moment at the interface between the column and the footing:



$$M_u = (397x1,85)x0,775x\frac{0,775}{2} = 220,6\ kN.m$$

Ensure that M_{cr} > M_u otherwise compression steel will be required.

$$M_{cr} = 0,15 \cdot f_{cu} \cdot b \cdot d^2$$

Cover to reinforcing = 75 mm for foundations.

d = 450 – 75 –25/2 = 362,5 mm, use 360 mm (We assume 25 mm bars hence 25/2 subtracted)

z = a_1.d

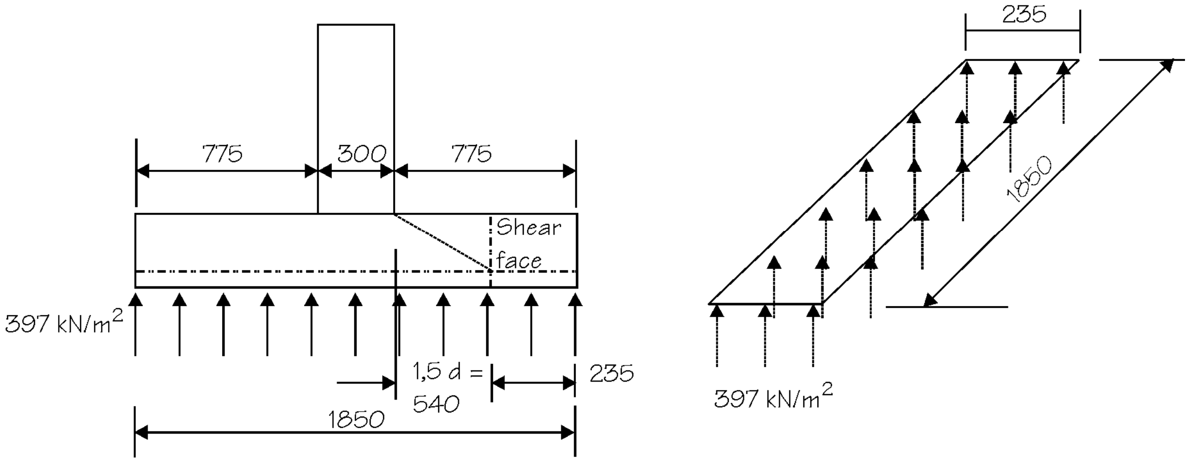
$$a_1 = \frac{1+\sqrt{1-\frac{5 \cdot M_u}{f_{cu} \cdot b \cdot d^2}}}{2} = \frac{1+\sqrt{1-\frac{5 \cdot 220,6x10^6}{25 \cdot 1850 \cdot 360^2}}}{2} = 0,95$$

$$A_s = \frac{M_u}{0,87 \cdot f_y \cdot z} = \frac{220,6x10^6}{0,87 \cdot 250_y \cdot (0,95 \cdot 360)} = 2966\ in\ 1850\ mm\ width$$

$$\therefore A_s = \frac{2966}{1,850} = 1603\ mm^2 / m\ width$$

Use R25 @ 300 mm (1636 mm^2) in both directions.

Test for Shear



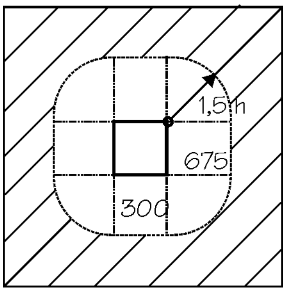
V_u = 0,235 x 1,850 x 397 = 172,6 kN

$$\text{Shear stress} = \frac{V_u}{\text{Cross-sectional area}} = \frac{172{,}6x10^3}{1850 \cdot 360} = 0{,}259\ MPa$$

If % steel = $\frac{100 \cdot A_s}{b \cdot d} = \frac{100 \cdot 1636}{1850 \cdot 360} = 0{,}246\%$

With f_{cu} = 25 MPa and % steel = 0,246%, the design table gives a v_{cr} = 0,38 MPa which is greater than the induced shear stress.

Punching Shear



Punching circumference = 4 x 300 + π x D
= 4 x 300 + π x 3 h
= 4 x 300 + π x 3 x 450
= 5441 mm

Hatched area will tend to stay behind as column tries to punch through.

Hatched area = $1850^2 - (300x300 + 4x300x675 + \pi x675^2)$
= $1{,}091 \times 10^6\ mm^2$
= $1{,}091\ m^2$

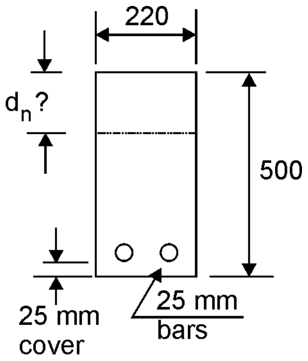
$V_{punching}$ = 1,091 x 397 = 433,13 kN

Punching stress $\frac{V_{punching}}{Circumference \cdot d} = \frac{433{,}13x10^3}{5441 \cdot 360} = 0{,}221\ MPa < 0{,}38\ MPa$

REINFORCED CONCRETE – ADDITIONAL PROBLEMS

Question1:

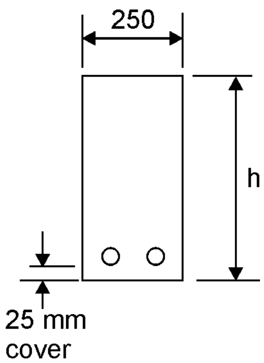
Determine the **neutral axis depth** as well as the **resistance moment** of the concrete beam with the following cross-sectional dimensions. Concrete is a grade 30 MPa, i.e., f_{cu} = 30 MPa, bars are high tensile with a yield stress of 450 MPa. (Answer d_n = 146 mm, M = 150,1 kN.m)



Question 2:

A simply supported 250 mm wide beam, spans 8 m and carries the following serviceability (unfactored) loads. Determine the required **height, h**, and the **tensile steel**. (One possible set of answers: h = 575 mm, A_s = 1512, i.e., 2 x 25 mm + 2 x 20 mm bars)
DL = 10 kN/m
LL = 12 kN/m

Concrete grade = 25 MPa
Reinforcing stress = 450 MPa



Question 3:

You are asked to design a simply supported concrete slab over a garage with dimensions 4,5 m x 6 m. The floor carries a live load of 1,5 kN/m^2 and an additional dead load from the floor finish of 1,2 kN/m^2. Use a grade 25 MPa concrete and high tensile steel, f_y = 450 MPa with the cover to the steel = 20 mm. Determine the thickness of the slab and the reinforcing in both directions. (Possible answers: Thickness = 170 mm, w_u = 8,736 kN/m^2, M_{ux} = 17,5 kN.m, M_{uy} = 9,73 kN.m, A_{sx} = 331 mm^2/m, A_{sy} = 198 mm^2/m)

Question 4:

A 250 x 600 mm concrete beam with a concrete strength of 25 MPa has an ultimate applied moment of 330 kN.m. Yield stress of bars = 450 MPa, cover to bars = 25 mm. Determine the tension and compression steel. (Answers: A'_s = 369 mm^2, A_s = 2018 mm^2)